THE MATHEMATICAL PROGRAM
AT THE
NAVAL PROVING GROUND

U. S. NAVAL PROVING GROUND
DAHLGREN, VIRGINIA

The Mathematical Program

at the

Naval Proving Ground

by

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Computation and Exterior Ballistics Laboratory

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I would like to give you some idea of how the mathematicians at Dahlgren are organized, what type of work they do, and a few examples of things that have been done there in the recent past.

The first slide shows the numbers and grades of mathematicians employed at the Naval Proving Ground and where they are located. The Computation & Exterior Ballistics Laboratory is one of three at Dahlgren, and is organized as shown at the top of the slide. Not all the branches are listed; just those containing mathematicians. As you can see, the greater share are in the Programming Branch, and in the two branches concerned with ballistic problems.

The main computing facility used by this staff is, of course, the NORC. The problems that these people work on and the fiscal support for their work come from various sources. Perhaps the bulk of the work is concerned with ballistics problems, for which the analysis is done in the Missile Development and Evaluation Branch and the Theory and Analysis Branch, and any programming necessary, in the Programming and Coding Branch, the other branches shown being used for occasional consulting. Fiscal support comes mostly from Bureau of Ordnance, and charges are made to the various Bureau research and development projects. Mathematical research and development of numerical techniques in ballistics are generated within the ballistics branches in response to the need for increased precision.

Most of the recent efforts of the Mathematical Physics Branch has been invested in the numerical solution of a problem in hydrodynamics which will be described in the sequel. Fiscal support for this work has been from so-called foundational research funds. Some support for solution of a problem concerning resistance to a ship from water waves (of interest to BuShips) is expected from ONR.

The Applied Mathematics Branch has divided its efforts between projects of varying sizes undertaken at the request of other branches or of agencies outside of the Naval Proving Ground, and a continuing program of research in numerical methods. Of the latter, one phase has received Bureau of Ships support, the remainder being done under foundational research funds.

The Programming Branch provides programming and coding services to the other branches in the Laboratory and to outside activities primarily under BuOrd. This branch is also charged with a responsibility for research in advanced programming methods as well as for the development of a subroutine library.

I hope the foregoing has given you some idea of the organization and support of our mathematicians. I would now like to describe some examples of recent and current work done in the various branches.

The first example, in the field of mathematical ballistics, is a significant piece of work reported by C. J. Cohen and D. Werner in NAVORD 5133. The rigid body equations of motion of a spinning projectile with four fold rotational symmetry were formulated, account being taken of the dependence of the aerodynamics force-and-moment system on the angle of roll (notice Slide 2). The dimensionless coefficients are assumed to be functions of yaw and roll angle (but independent of Mach number, Reynolds number, etc.) and are expanded in Fourier series in the roll angle. In place of the Eulerian angle system of coordinates, use was made of classical quaternions, a technique which has several advantages for the subsequent numerical computation. The numerical integration of these equations of motion has been coded for the NORC and successfully applied to real missile configurations. This technique is being used again in calculations for the Vanguard program.

A second example which applies to ballistics work concerns the fitting of polynomials to data by least squares criteria, an old technique in numerical analysis, but one which is continually receiving embellishments. A problem of interest in ballistics work is the determination of the optimum order of the polynomial to be used to adequately represent the given data without being unduly influenced by random errors in the data. Mr. J. E. Barker has used polynomials orthogonal with respect to a given weight function over the discrete set of data points. The coefficients are given by recurrence relations and the order of the polynomial selected by statistical tests (such as the F test or t test) based on the estimated standard deviation of the data at each point. Estimates of the variances of the fit and of derivatives of the fit are developed. A program has been prepared for the NORC incorporating these techniques for sets of equally spaced data points, and is being extended to unequally spaced data.

A third example from ballistics concerns a trajectory problem. The usual trajectory problem reduces to the numerical solution of a system of ordinary differential equations. In recent months we have been faced with a variation of this problem. The analysis of a certain aircraft maneuver has led to a system of differential equations with an additional equation of constraint which serves to determine an extra variable. At present we are experimenting with a "Rule of False Position" method of iteration. At each step of the integration a sequence of trial values of the extra variable is used, the integration-step being repeated for each trial value, until the constraint equation is satisfied. The iteration equations for a type problem are shown in Slide 3.

The second group of examples concern work done in the Applied Mathematics Branch.

One of the first problems given to the NORC was the numerical solution of the free boundary problem shown in Slide 4. The mathematical model is that of the irrotational flow of an ideal incompressible fluid past a pair of disks, with axial symmetry. The differential equation and boundary conditions were replaced by a difference system which was solved for a given trial boundary by the method of SOR. Different techniques, none successful, were tried to provide automatic improvement of the trial boundary. This work was reported in NPG 1413.

Under the leadership of David M. Young, who devoted part of his time to NPG while at Maryland University, the Applied Mathematics Branch investigated iterative methods for the solution of elliptic difference systems. This work was partly supported by the Bureau of Ships which was interested in such systems as approximations to reactor problems. One result of this study was a generalization of Young's classical paper on the Successive Overrelaxation method to cases in which a block of unknowns rather than a single unknown is corrected at each step of the iteration, an example being the technique of line relaxation, which was then being programmed for reactor problems. It was found that under certain conditions on the difference system, convergence was faster with block relaxation than with point relaxation, also, the theory applies with a larger class of difference equations. A paper by Arms, Gates and Zondek reporting these results will appear in the Journal of SIAM.

An interesting example of a large numerical problem was obtained from the theory of Magnus forces due to boundary layer formation on a slowly spinning cylinder with a small yaw angle. A set of 26 linked ordinary differential equations was derived by Dr. Kelly of Naval Ordnance Test Station and sent to NPG for numerical solution. When taken in proper order the set reduced to the Blasius equation and 18 simultaneous first order linear systems, each system containing two or three equations with conditions split between the origin and infinity. A successful method was devised for the numerical solution of these systems. In the preliminary analysis of this system, an estimate of the error in the solution of the Blasius equation due to moving the boundary condition to a finite point, was discovered by L. A. Rubel and reported in the Quarterly of Applied Mathematics. It is interesting to note that after preliminary results from NORC tipped us off, we were able to show that one of the equations was almost certainly incorrectly set. This work was reported by Gates and Arms in NPG 1457.

The Mathematical Physics Branch, under the direction of A. V. Hershey, has lately been occupied in the development of techniques for a numerical solution of a model of the growth of a cavity in an ideal incompressible fluid. A circular disk placed perpendicular to an infinite uniform flow is allowed to follow the flow and then is instantaneously stopped. Initial conditions on the flow potential have been selected on the basis of physical arguments, for example, that the pressure be everywhere positive except where separation of the flow is iminent where it is allowed to be zero. This gives a boundary problem for the initial potential field which is displayed in Slide 5. Numerical evidence indicates that a solution exists for this initial problem. The second part of the problem is to follow the flow through time, to determine in particular, the successive positions of the cavity boundary, which should form from the line OQ.

An integral equation over the boundary is to be employed rather than a grid in the flow field, as was used in the cavity problem mentioned earlier. It is, of course, not expected that a steady state will be reached, as no irrotational steady state solution with a cavity is known except flows involving a re-entrant jet or an infinite cavity.

As a by-product of this work high accuracy subroutines have been designed by the Mathematical Physics Branch for elementary functions and matrix algebra. The first category includes routines for incomplete elliptic integrals and Fresnel integrals. The second includes conjugate gradient, Gauss-Seydel and partitioning routines for linear systems and matrix inverses, and the Lanczos method for eigenvalues and vectors. The capacity of these routines is variable, generally they will handle matrices of order over one hundred.

Finally, it should be mentioned that some important work has been done at Dahlgren, on the NORC, by people from other organizations. Of principal interest is the program of computation of orbits for the earth, the moon, Mars and minor planets over the period 1920-2000 AD, under the direction of Dr. Paul Herget, of Cincinnati Observatory, and supported by Office of Naval Research. A letter written to the National Science Foundation by Dr. Herget stated, "During May 1956, we used 9 hours of running time with this program and completed more computations than had ever before been done at one time in the history of Astronomy. We completed the accurate perturbations of nearly 100 selected minor planets for an average of more than 50 years each."

Considerable time on NORC has been devoted to reactor design problems sponsored by the Bureau of Ships. I will not take time to describe this work as it has been covered by the Model Basin representative.

Other examples of problems solved on the NORC could be given, time permitting, but I think it would be best to stop here and allow time for any questions from the floor.

Location

Grade	Computation & Exterior Ballistics Laboratory (One GS-14)					Other Labs.	Total
14	Comp. Division (One GS-14)		Exterior Ballistics Division				2
	Appl. Math.	Progr. & Coding	Math. Physics	Theory & Analysis	Missile Develop. & Eval.		
13	1	1	1	1			4
12	2	1	1	3	2		9
11		4		4	2		10
9		2	1	3	1	1	8
7	1	8		3	5	1	18
5		6		3			9
Branch Totals	4	22	3	17	10		60

Figure 1 - Organization and Grades of Mathematicians at NPG

FIN STABILIZED MISSILE FORCE AND MOMENT SYSTEM FOR CASE OF NOSE DOWN YAW AND RIGHT HANDED SPIN

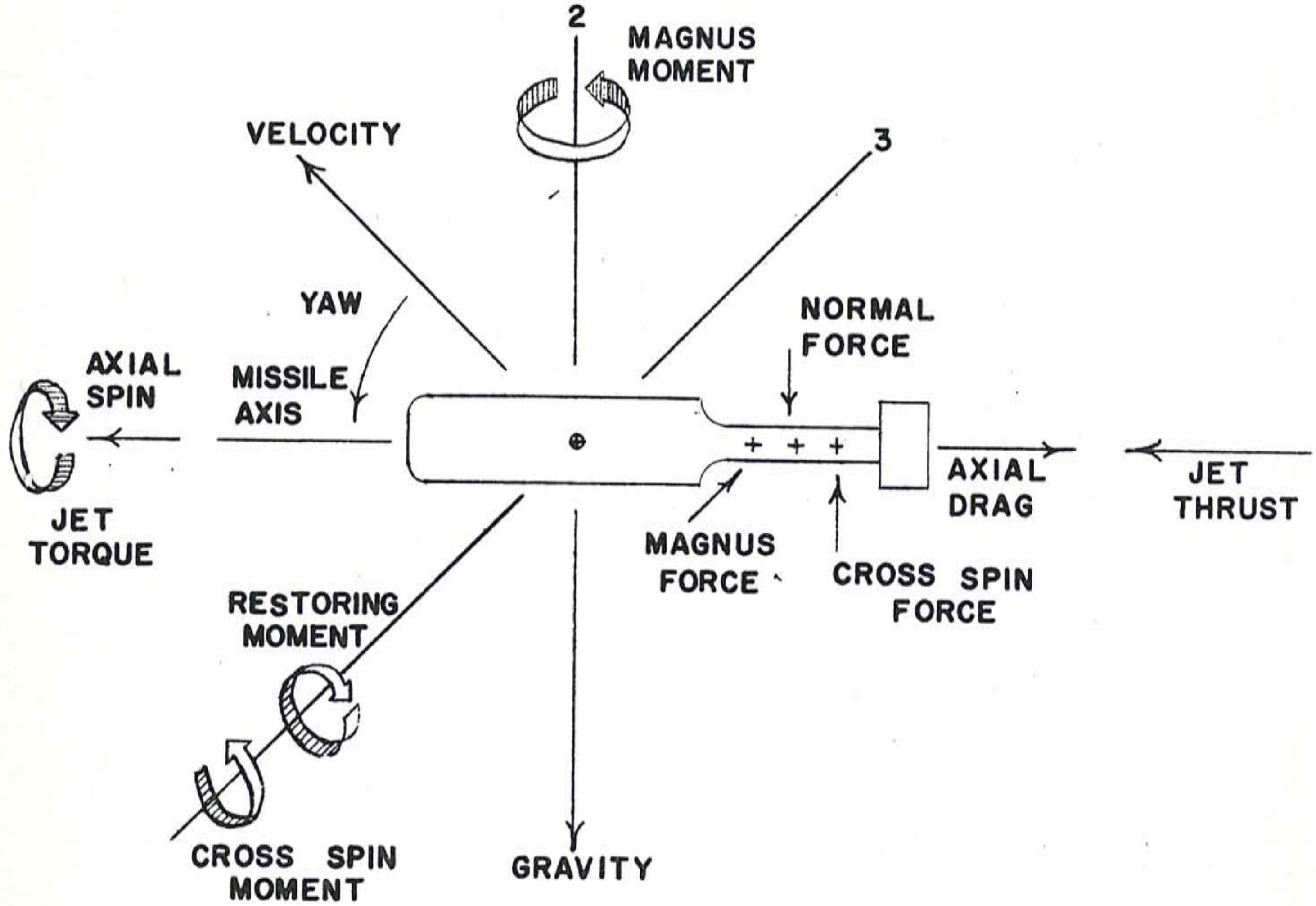

FIG. 2. RIGID BODY EQUATIONS

1. Equations of Motion

$$\dot{x} = f(x, y, \delta)$$

$$\dot{y} = g(x, y, \delta)$$

$$F(x, y, \delta, t) = 0$$

2. Iteration for integration from t_o to t_1.

Given: $x(t_o)$, $y(t_o)$, $\delta(t_o)$; $x^{(n)}(t_1)$, $y^{(n)}(t_1)$, $\delta^{(n)}(t_1)$.

$$(1)\quad \delta^{(n+1)}(t_1) = \delta^{(n)}(t_1) - F_n\left[\frac{\delta^{(n)}(t_1) - {}^{(n-1)}(t_1)}{F^{(n)} - F^{(n-1)}}\right]$$

$$F^{(n)}(t_1) = F(x^{(n)}(t_1), y^{(n)}(t_1), \delta^{(n)}(t_1), t_1)$$

$$(2)\quad x^{(n+1)}(t_1) = x(t_o) + \frac{\Delta t}{2}\Big[f(x(t_o), y(t_o), \delta(t_o)) + f(x^{(n+1)}(t_1), y^{(n)}(t_1), \delta^{(n+1)}(t_1))\Big]$$

$$(3)\quad y^{(n+1)}(t_1) = y(t_o) + \frac{\Delta t}{2}\Big[g(x(t_o), y(t_o), \delta(t_o)) + g(x^{(n+1)}(t_1), y^{(n+1)}(t_1), \delta^{(n+1)}(t_1))\Big]$$

Figure 3 - Aircraft Manoeuvre Equations

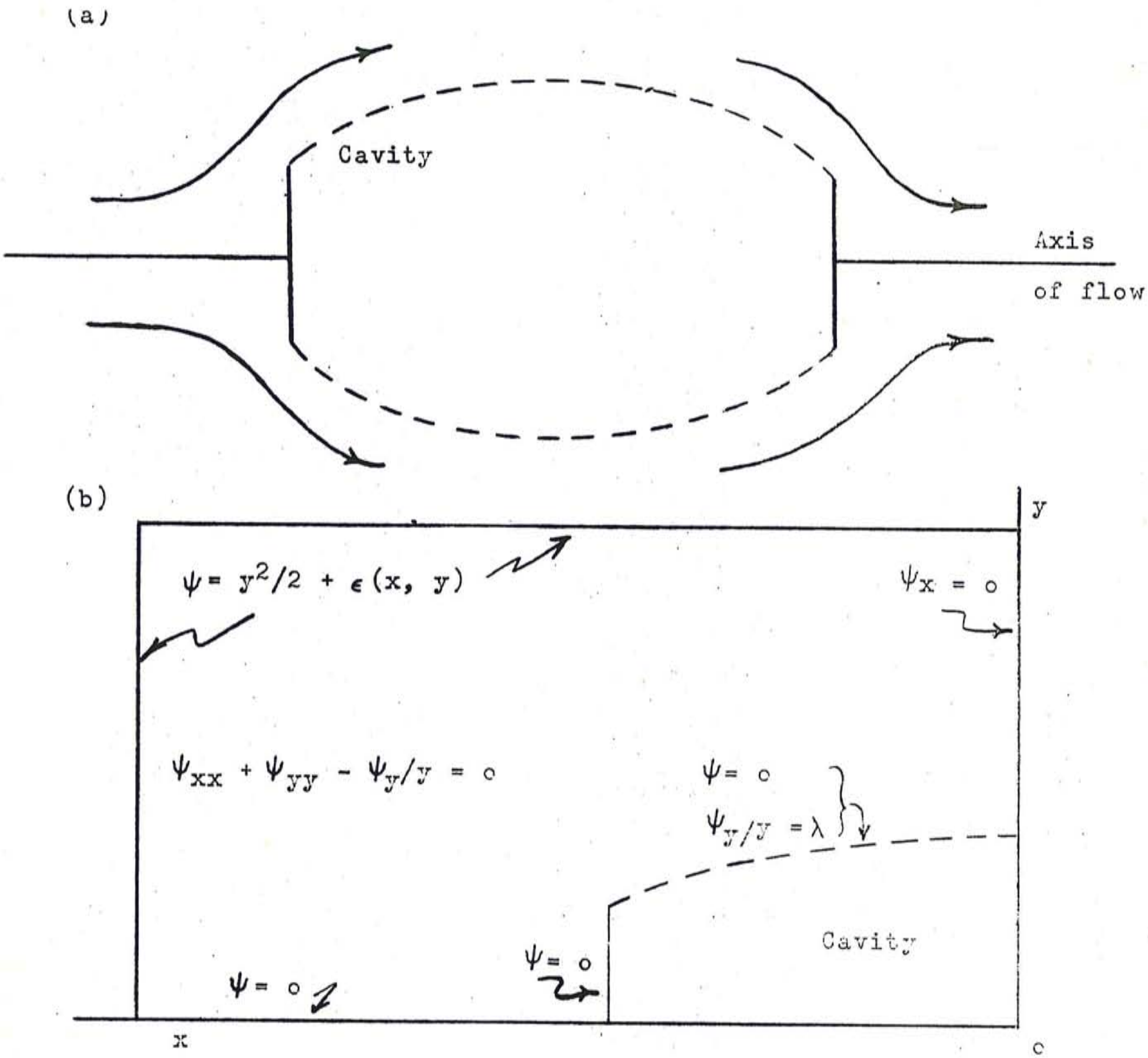

Figure 4 - Cavitational Flow

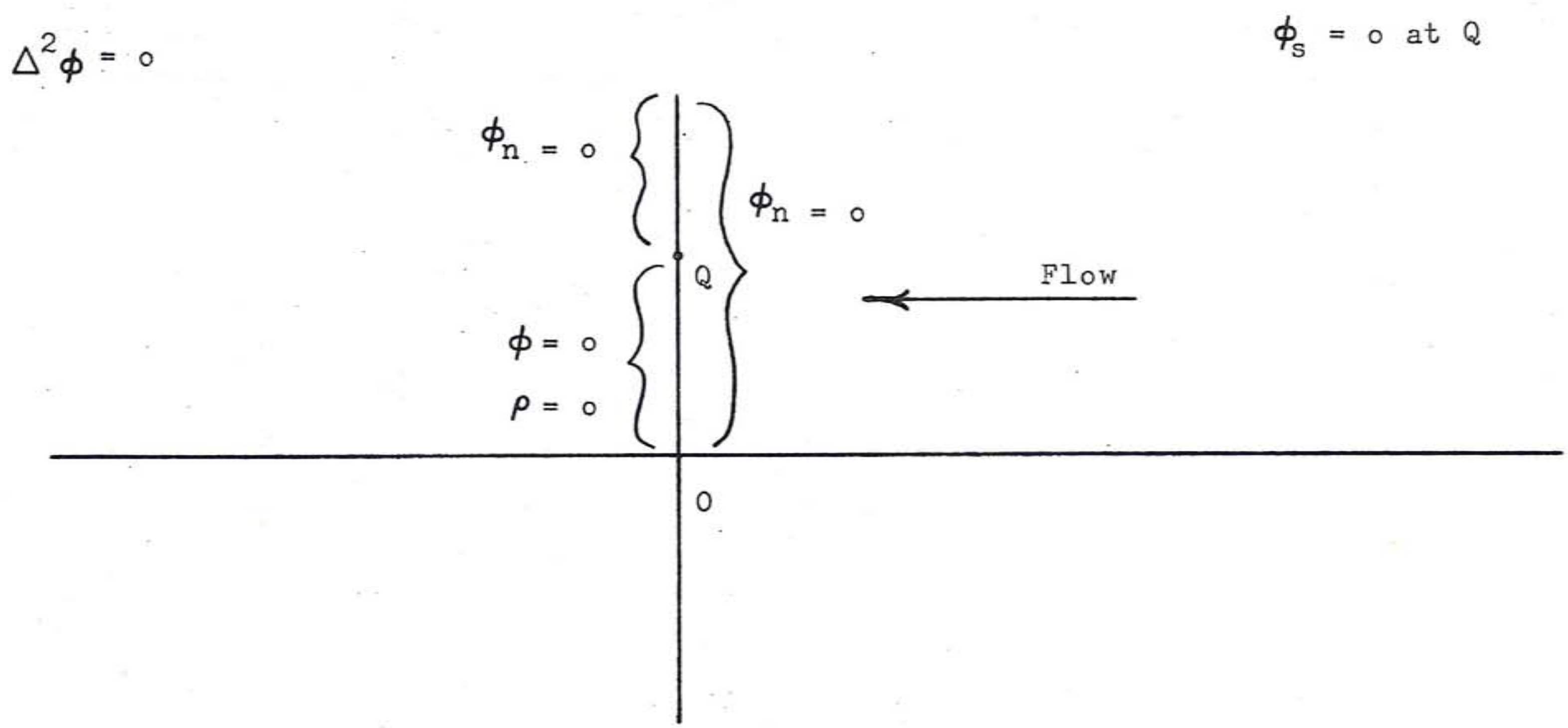

Figure 5 - Growth of a Cavity, Initial Conditions.